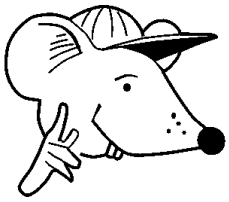
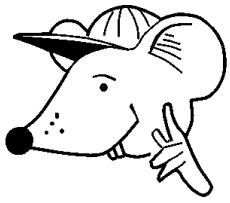
Back to Menu



MATHEMATICS



N.S. Yr. 6 P.21 EXTENSION

Recognising square numbers, cube numbers, square roots and prime factors.

Equipment

Paper, pencil, ruler.

MathSphere

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

Concepts

The term '**square number**' needs to be recognised and understood.

7 x 7 can be written as 7^2 This is pronounced, " **seven squared**".

Many children are confused with this and believe that the squared sign (^{2}) means multiply by two - which is incorrect. A reminder might be needed.

If 7^2 **= 7 x 7 = 49,** then the square root of **49** is **7.**

The square root sign is $\sqrt{\ }$ so $\sqrt{49}$ is **7**

The cube of a number is written 3^3 **.** This is pronounced 'three cubed**.**'

3^3 equals **3 x 3 x 3** which equals **27.**
10^3 equals **10 x 10 x 10** which equals **1 000**

Multiples of numbers are numbers which are produced by multiplying that number by another whole number. Eg **4, 6, 8** are multiples of **2**.

The **factors** of a number (eg 24) are those numbers which divide exactly into it.
The factors of **24** are **1, 2, 3, 4, 6, 8, 12, 24.**
Every whole number has at least two factors - one and the number itself**.**

A number which only has two factors (one and itself) is called a **prime number.**
The first prime numbers are **2, 3, 5, 7, 11, 13, 17** and **19.**
(Note that **1** is not usually considered as a prime number.)

To decide whether a number is prime, a check has to be made as to whether it has any factors.

Try dividing by the first few prime numbers, **2, 3, 5, 7, 11.** See examples on sheets.

(See also sheets on multiples and tests of divisibility.)

Prime factors of a number are the factors which are **also prime numbers.**
Eg the factors of **12** are **1, 2, 3, 4, 6** and **12.**
Of these factors **2** and **3** are both prime and are therefore prime factors**.**
12 can be expressed in terms of prime factors by using the prime factors more than once: **12 = 2 x 2 x 3**

Square numbers

Example: $3^2 + 4^2 = (3 \times 3) + (4 \times 4) = 9 + 16 = 25$

This is pronounced 'three squared plus four squared'.

The square of the number must be worked out first, then the two answers added.
Try these:

1. $3^2 + 5^2 =$ **2.** $4^2 + 6^2 =$ **3.** $7^2 + 7^2 =$

4. $6^2 - 4^2 =$ **5.** $10^2 - 7^2 =$ **6.** $9^2 - 2^2 =$

7. Which of the numbers in the box below are square numbers?

169	**200**	**1 000**	**2025**	**18**	**9**

8. What is the square root of 169?

Some numbers are equal to the sum of two square numbers.
For example 34 = $3^2 + 5^2$

Find the two square numbers which make up these totals:

9. 74

10. 85

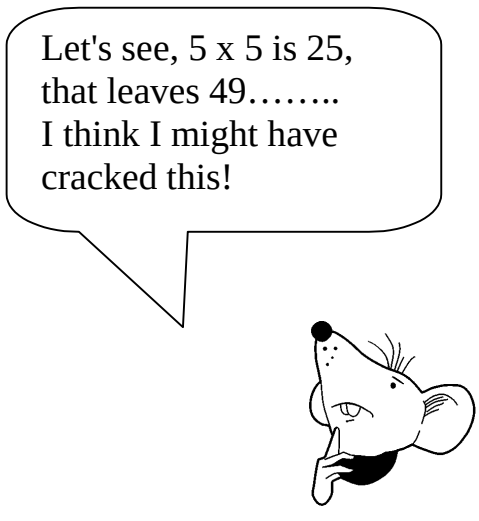
11. 45

12. 181

13. 52

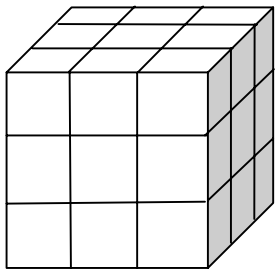
14. 113

15. 40



 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

The cube of a number



Above is a 3 x 3 x 3 cube.

Altogether there are 27 cubes, because 3 x 3 x 3 = 27

3 x 3 x 3 can be written 3^3 for short. We say ' three cubed'.

1. How do you say 5^3 ? Write the words down.

2. How do you say 7^3 ? Write the words down.

The cube of the number 5 is 5^3 which equals 5 x 5 x 5 which is 125.

(**Note the cube sign does not mean multiply by three!)**

The cube of 6 equals 6 x 6 x 6

In the same way write down what these mean (do not work out the answer):

3. the cube of 8

4. the cube of 4

5. the cube of 10

6. the cube of 9

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

The cube of a number

Work out the answers to these numbers:

1. 2^3

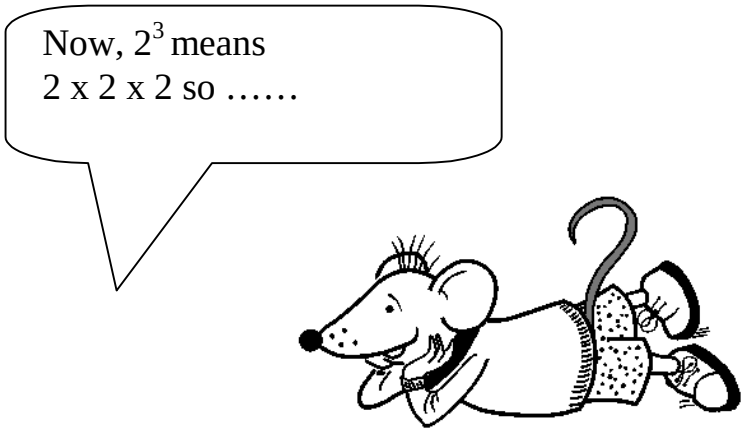
2. 3^3

3. 4^3

4. 1^3

5. 5^3

6. 10^3



You may have noticed that cube numbers start getting pretty big, pretty quickly.

Often you will use a calculator to work out cubes.

Example; to find 5^3 you can press 5 x 5 x 5 =
(even quicker, you can press 5 x = = on some calculators or
5 x x = = on other calculators)

Find which way your calculator works and then work out, using the calculator these numbers:

7. the cube of 11 **8.** 9^3 **9.** the cube of 13 **10.** 6^3

Some scientific calculators might have a key $\boxed{x^y}$

In this case x is the number and y is the cube so, to find 7^3 press

7 $\boxed{x^y}$ 3 = and 343 should display. Good luck!!

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

Square roots

You should be able to recognise the **square roots** of certain numbers.

Remember if 5 x 5 = 25 then the square root of 25 is 5.

Write down the square roots of these numbers:

1. 36 **2.** 100 **3.** 121 **4.** 81 **5.** 196

All these have whole number square roots, but often the square root of a number has a decimal fraction.

For example, using a calculator, the square root of 3 is 1.73 205 08

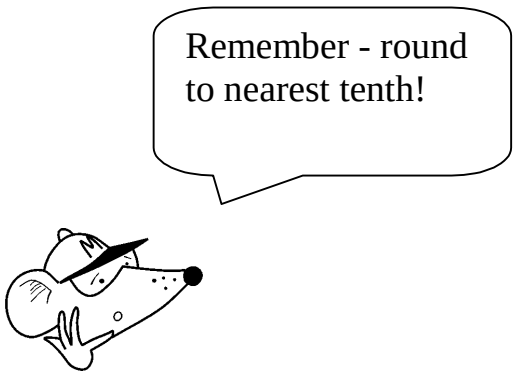
Remember, you only have to type in the number and press the square root sign (√) on the calculator.

It is usual to round off a square root like this to the nearest tenth.

To do this look at the second number after the decimal point. If it is 5 or more round up, if it is less than five round down.

1.7**3**2 050 8 - the second number after the decimal is 3 so round down, making the square root of 3 as 1.7 to the nearest tenth.

Using a calculator, find the square roots of these numbers and round them to the nearest tenth.



6. 24

7. 2

8. 7

9. 80

10. 22

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

Cube roots

2 x 2 x 2 = 8 so, the cube root of 8 is 2
3 x 3 x 3 = 27 so, the cube root of 27 is 3

Work out the cube roots of these numbers:

1. 64 **2.** 1 **3.** 125 **4.** 216 **5.** 1 000

Only a very few small numbers have a cube root which is an exact whole number.
One way to find the cube root of a number is by:

TRIAL AND IMPROVEMENT.

Example: find the cube root of 60 correct to the nearest tenth.

Try:	4 x 4 x 4 = 64	4 is too big
	3 x 3 x 3 = 27	3 is too small
	3.7 x 3.7 x 3.7 = 50.653	3.7 is too small
	3.9 x 3.9 x 3.9 = 59.319	3.9 is too small (but only just!)
	3.91 x 3.91 x 3.91 = 59.776	3.91 is still too small, but very close
	3.92 x 3.92 x 3.92 = 60.236	3.92 is too big, just!

The cube root of 60 is between 3.91 and 3.92.
The cube root of 60 is 3.9 to the nearest tenth.

Use your calculator and trial and improvement methods to find the cube roots of these numbers, to the nearest tenth:

6. 12

7. 20

8. 30

9. 66

10. 40

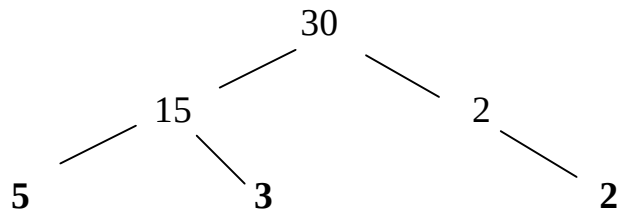
Prime factors

The **prime factors** of a number are the **factors which are also prime numbers.**
Reminder: 1 is not usually thought of as a prime number so the first few prime numbers are 2, 3, 5, 7, and 11

How to find the prime factors of a number eg 30:

Start by seeing if the number is divisible by two.
If the number is not divisible by two then test for 3, then 5 and then 7, going through all the prime numbers, until only prime numbers are left.

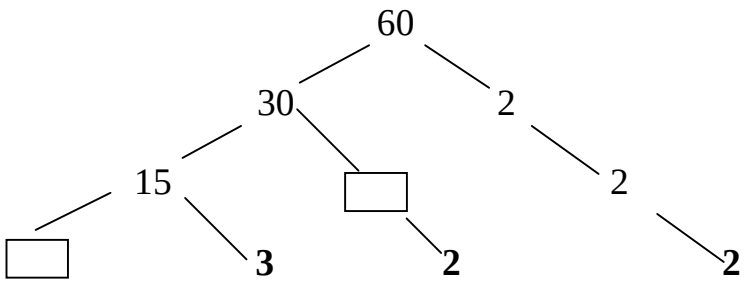
Draw a 'factor tree' like the one below to show the divisions:



The prime factors of 30 are 2 x 3 x 5 = 30

Often the prime factors of a number will include the same factor several times, especially 2.

1. Fill in the gaps in this factor tree of 60



The prime factors of 60 are 2, 2, 3, and 5, because 2 x 2 x 3 x 5 = 60

Find the prime factors of these numbers:

2. 26 **3.** 50 **4.** 16 **5.** 27 **6.** 40

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

Prime factors

Draw factor trees to find the prime factors of the following numbers. You might find isometric paper helpful to draw out the diagrams:

1. 24 **2.** 55 **3.** 30 **4.** 28 **5.** 81

Trial and improvement using a calculator

Example: Use trial and improvement methods to find two consecutive numbers which, when multiplied together, come to 1 260.
Set it out like this:

Try 1	24 x 25 = 600	too small
Try 2	38 x 39 = 1 482	too large
Try 3	34 x 35 = 1 190	too small, but close
Try 4	35 x 36 = 1 260	spot on!!! ☆

Use trial and improvement methods to find two consecutive numbers which, when multiplied together, come to:

6. 210 **7.** 7 482 **8.** 2 070 **9.** 306 **10.** 9 702

Square, cube numbers and square roots using a calculator

Find the value of:

11. 4^3 **12.** 7^3 **13.** 15^3 **14.** 3^3 **15.** 11^3

These numbers are equal to the sum of two square numbers
Eg $34 = 3^2 + 5^2$
Find the two square numbers for each of these:

16. 181 **17.** 113 **18.** 104 **19.** 221 **20.** 145

Find the square root of these numbers, rounding to the nearest tenth:

21. 37 **22.** 17 **23.** 79 **24.** 60 **25.** 11

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

Prime factors

Draw factor trees to find the prime factors of the following numbers. You might find isometric paper helpful to draw out the diagrams:

1. 42 **2.** 66 **3.** 70 **4.** 82 **5.** 28

Trial and improvement using a calculator

Example: Use trial and improvement methods to find three consecutive numbers which, when multiplied together, come to 13 800.
Set it out like this:

Try 1	15 x 16 x 17 = 4 080	too small
Try 2	20 x 21 x 22 = 9 240	still too small
Try 3	31 x 32 x 33 = 32 736	too big
Try 4	24 x 25 x 26 = 15 600	too big, but close
Try 5	23 x 24 x 25 = 13 800	brilliant!! ☆

Use trial and improvement methods to find three consecutive numbers which, when multiplied together, come to:

6. 720 **7.** 74 046 **8.** 1 320 **9.** 140 556 **10.** 24

Square, cube numbers and square roots using a calculator

Find the value of:

11. 8^3 **12.** 13^3 **13.** 55^3 **14.** 21^3 **15.** 42^3

These numbers are equal to the sum of two square numbers
Eg $34 = 3^2 + 5^2$
Find the two square numbers for each of these;

16. 346 **17.** 293 **18.** 58 **19.** 169 **20.** 5

Find the square root of these numbers, rounding to the nearest tenth:

21. 57 **22.** 93 **23.** 101 **24.** 500 **25.** 200

 P.O. Box 1234 Worthing BN13 2UJ www.mathsphere.co.uk

Answers

Page 3
1. 34 **2.** 52 **3.** 98 **4.** 20 **5.** 51 **6.** 77 **7.** 169, 2025, 9
8. 13 **9.** 5^2 and 7^2 **10.** 6^2 and 7^2 or 2^2 and 9^2 **11.** 3^2 and 6^2
12. 9^2 and 10^2 **13.** 4^2 and 6^2 **14.** 7^2 and 8^2 **15.** 2^2 and 6^2

Page 4
1. five cubed **2.** Seven cubed **3.** 8 x 8 x 8 **4.** 4 x 4 x 4 **5.** 10 x 10 x 10 **6.** 9 x 9 x 9

Page 5
1. 8 **2.** 27 **3.** 64 **4.** 1 **5.** 125 **6.** 1 000 **7.** 1 331 **8.** 729 **9.** 2 197 **10.** 216

Page 6
1. 6 **2.** 10 **3.** 11 **4.** 9 **5.** 14 **6.** 4.9 **7.** 1.4 **8.** 2.6 **9.** 8.9 **10.** 4.7

Page 7
1. 4 **2.** 1 **3.** 5 **4.** 6 **5.** 10 **6.** 2.3 **7.** 2.7 **8.** 3.1 **9.** 4.0 **10.** 3.4

Page 8
1. 2, 5 **2.** 2, 13 **3.** 2, 5, 5 **4.** 2, 2, 2, 2 **5.** 3, 3, 3 **6.** 2, 2, 2, 5

Page 9
1. 2, 2, 2, 3 **2.** 5, 11 **3.** 2, 3, 5 **4.** 2, 2, 7 **5.** 3, 3, 3, 3 **6.** 14 x 15 **7.** 86 x 87
8. 45 x 46 **9.** 17 x 18 **10.** 98 x 99 **11.** 64 **12.** 343 **13.** 3 375 **14.** 27 **15.** 1 331
16. $9^2 + 10^2$ **17.** $7^2 + 8^2$ **18.** $2^2 + 10^2$ **19.** $10^2 + 11^2$ **20.** $1^2 + 12^2$ or $8^2 + 9^2$
21. 6.1 **22.** 4.1 **23.** 8.9 **24.** 7.7 **25.** 3.3

Page 10
1. 2, 3, 7 **2.** 2, 3, 11 **3.** 2, 5, 7 **4.** 2, 41 **5.** 2, 2, 7 **6.** 8 x 9 x 10 **7.** 41 x 42 x 43
8. 10 x 11 x 12 **9.** 51 x 52 x 53 **10.** 2 x 3 x 4 **11.** 512 **12.** 2 197 **13.** 166 375
14. 9 261 **15.** 74 088 **16.** $11^2 + 15^2$ **17.** $2^2 + 17^2$ **18.** $3^2 + 7^2$ **19.** $5^2 + 12^2$
20. $1^2 + 2^2$ **21.** 7.5 **22.** 9.6 **23.** 10.0 **24.** 22.4 **25.** 14.1