

MATHEMATICS



N.S. Yr. 6 P.81

**Make and investigate general statements
about numbers and shapes**

Equipment

Paper, pencil, calculator, angle measurer.

MathSphere

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Concepts

Children should be able to spot a relationship between variables in a situation and begin to give a formula to represent the situation.

Examples can be taken from any aspect of the syllabus, but may typically include the following:

Finding decimals whose values lie between two given decimals.

Multiplying by a number in stages (e.g. to multiply by 25 first multiply by 100 and then halve the answer twice).

Relationships between square numbers and triangle numbers.

Dividing by a half or quarter makes the number twice/four times as big.

Sum of the angles of a triangle.

Develop rules for sequences, including producing a formula for the general term.

1. Give some numbers that can go in the box:

$$0.37 < \square < 0.40$$

2.

Let's try adding three consecutive numbers.



Look at these sums and compare the answers with the middle number in the sum.
What do you notice?

$$6 + 7 + 8 \quad 12 + 13 + 14 \quad 22 + 23 + 24$$

3. Now try this with three consecutive numbers from one of the times tables.

Here are some examples:

$$8 + 12 + 16 \quad 25 + 30 + 35 \quad 14 + 21 + 28 \quad 18 + 24 + 30$$

What do you notice?

4. Can you try it with decimals or negative numbers?

Here are some examples:

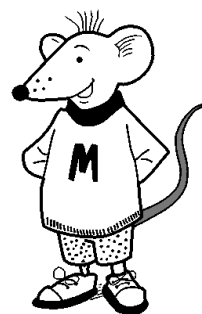
$$3.4 + 3.5 + 3.6 \quad 5.4 + 5.7 + 6.0 \quad -4 + -3 + -2$$

What do you notice?

Can you find an example that does not work?

1.

Have you thought about taking short cuts when doing sums?



For example, to multiply by **50**, you could multiply by **100** and divide by **2**.

Here is an example:

What is 34×50 ? First multiply **34** by **100**. $\Rightarrow 34 \times 100 = 3\,400$
 Then divide **3 400** by **2**. $\Rightarrow 3\,400 \div 2 = 1\,700$

So, $34 \times 50 = 1\,700$!

Brilliant!



Can you use similar ideas to work out these sums?

- a. 26×50 b. 68×50 c. 48×25 d. 36×25 e. 44×25
 f. $2\,300 \div 50$ g. $8\,000 \div 50$ h. $12\,000 \div 25$ i. $900 \div 25$

2. Here are the first 10 triangle numbers:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

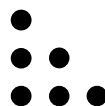
Look at pairs of **consecutive** numbers such as **15, 21** and add them up.

What type of number is the answer?

Try other pairs of triangle numbers.

What do you notice?

Can you complete this diagram for $3 + 6$ to show why this works?



1. Use a calculator to work out these sums:

$14 \div 0.5$ $23 \div 0.5$ $9 \div 0.5$ $44 \div 0.5$ $38 \div 0.5$ $124 \div 0.5$

What effect does dividing by a half (0.5) have on the number?

2. Use a calculator to work out these sums:

$5 \div 0.25$ $12 \div 0.25$ $20 \div 0.25$ $15 \div 0.25$ $40 \div 0.25$

What effect does dividing by a quarter (0.25) have on the number?

3. Can you predict the effect that dividing by 0.1 and 0.2 will have on a number. Try it to see if you are right.

4. Use a calculator to answer the following questions:

a. 3.5×10 b. 0.73×10 c. 5.2×10 d. 0.03×10 e. 12.34×10
f. 4.65×10 g. 3.8×10 h. 0.08×10 i. 45.12×10 j. 0.46×10

What happens to a decimal number when it is multiplied by 10?

5. Use a calculator to answer the following questions:

a. 7.3×100 b. 0.28×100 c. 3.9×100 d. 0.07×100 e. 7.36×100
f. 2.8×100 g. 12.63×100 h. 0.26×100 i. 15.62×100 j. 0.11×100

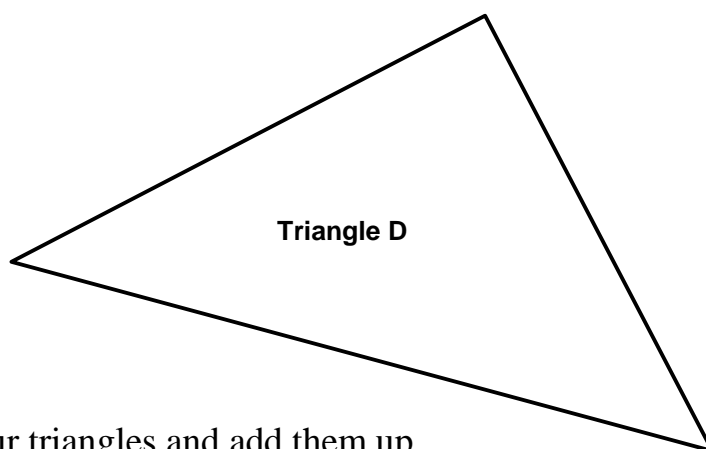
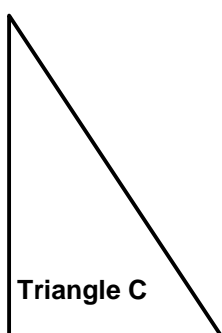
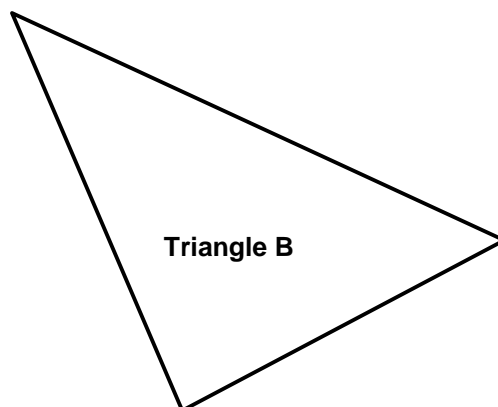
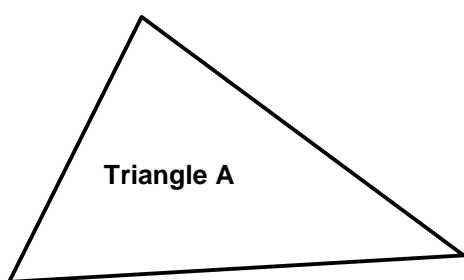
What happens to a decimal number when it is multiplied by 100?

6. From what you have already discovered, can you say what will happen to a decimal number when it is multiplied by 1 000 ?

Too many dots and zeroes for my brain!



1.



Measure the three angles in the four triangles and add them up.

Put your results in this table:

Triangle	First Angle	Second Angle	Third Angle	Total
A				
B				
C				
D				

What do you notice about the total of the angles in each triangle?

2. Now try the same with some quadrilaterals.

Draw some quadrilaterals, measure the four angles on each and add them up.

What do you notice? Can you present your results in a table?

1.

We could say a square is a rectangle with all four sides equal.

That's true!



What could you say about these shapes?

- a. rectangle b. equilateral triangle c. kite d. trapezium
e. parallelogram f. isosceles triangle g. regular hexagon
2. If **d** is the number of days in a week and **n** is the number of weeks, can you write a formula for the number of days in **n** weeks?
3. Write a formula for the area **A** of a rectangle if the length is **L** cm and width is **W** cm.
4. Write a formula for the perimeter **P** of the rectangle in question 3.
5. Write the general term for the **n**th term in this sequence:
2, 4, 6, 8, 10, 12, 14, ...
6. Write the general term for the **n**th term in this sequence:
4, 8, 12, 16, 20, 24, 28, ...
7. Soldiers are on parade.
k soldiers stand in one row.
There are **m** rows.
Each soldier wears **2** boots.

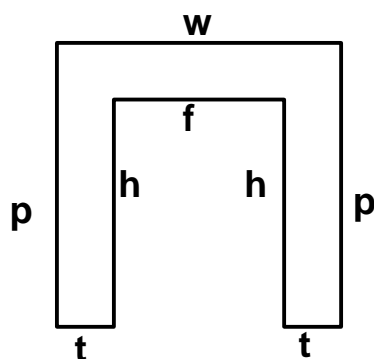
Do you think these boots are good enough for parade?



Write a formula for the number of boots **b** in the parade.

1. A bag of chips costs **85p**. What is the cost of **b** bags of chips (in pence)?
2. A pack of paper weighs **2.4 kg**. How much do **p** packs cost?

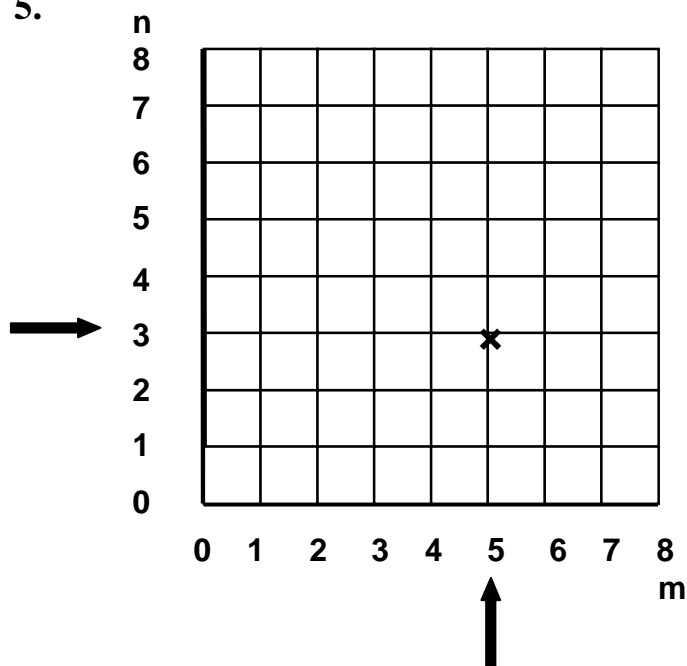
3.



What is the perimeter of this shape?

4. If a cake costs **c** pence and a bun costs **b** pence, what is the cost of a cake and a bun?
 What is the cost of three cakes and four buns?

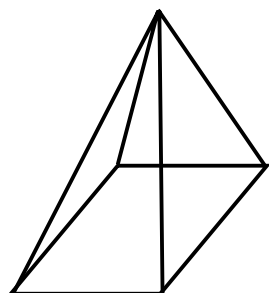
5.



Put crosses on this grid for all the pairs of numbers that add up to **8** ($m + n = 8$).

One has been done to start you off.

1. Here is square bases pyramid.



Count the number of **faces**, **edges** and **vertices**.

Now think about the **faces**, **edges** and **vertices** on other pyramids and put your results in this table.

Name	Edges on base (b)	Faces (f)	Edges (e)	Vertices (v)
Tetrahedron				
Square based pyramid				
Pentagonal based pyramid				
Hexagonal based pyramid				
Octagonal based pyramid				

What do you notice about the number of **faces** compared to the number of **edges on the base**?

Complete the formula to show this:

$$\mathbf{f} =$$

Now write down any other formula you can to show the relationships between the numbers in the table.

Answers

Page 3

1. There are many. Here are some possible answers:

0.38, 0.39, 0.375, 0.398

2. $6 + 7 + 8 = 21$, $12 + 13 + 14 = 39$, $22 + 23 + 24 = 69$

The total of the three numbers is three times the middle number.

3. $8 + 12 + 16 = 36$, $25 + 30 + 35 = 90$, $14 + 21 + 28 = 63$, $18 + 24 + 30 = 72$

The total of the three numbers is three times the middle number.

4. $3.4 + 3.5 + 3.6 = 10.5$, $5.4 + 5.7 + 6.0 = 17.1$ $-4 + -3 + -2 = -9$

The total of the three numbers is three times the middle number.

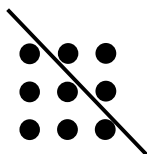
All examples taken from a multiplication table will work (even if it is the 0.2 times table or the -3 times table!)

Page 4

1. a. 1 300 b. 3 400 c. 1 200 d. 900 e. 1 100
 f. 46 g. 160 h. 480 i. 36

2. When consecutive triangle numbers are added, a square number is always obtained. Eg. $15 + 21 = 36$

Other pairs: $1 + 3 = 4$; $3 + 6 = 9$; $6 + 10 = 16$ etc



Answers (Contd)

Page 5

1. $14 \div 0.5 = 28$, $23 \div 0.5 = 46$, $9 \div 0.5 = 18$, $44 \div 0.5 = 88$,
 $38 \div 0.5 = 76$, $124 \div 0.5 = 248$

Dividing by a half doubles the number.

2. $5 \div 0.25 = 20$, $12 \div 0.25 = 48$, $20 \div 0.25 = 80$ $15 \div 0.25 = 60$,
 $40 \div 0.25 = 160$

Dividing by a quarter quadruples the number (makes it four times larger).

3. Dividing by 0.1 makes a number 10 times larger.
 Dividing by 0.2 makes a number 5 times larger.

4. a. $3.5 \times 10 = 35$, b. $0.73 \times 10 = 7.3$, c. $5.2 \times 10 = 52$,
 d. $0.03 \times 10 = 0.3$ e. $12.34 \times 10 = 123.4$, f. $4.65 \times 10 = 46.5$,
 g. $3.8 \times 10 = 38$ h. $0.08 \times 10 = 0.8$ i. $45.12 \times 10 = 451.2$,
 j. $0.46 \times 10 = 4.6$

When a number is multiplied by ten, all the digits move one place to the left.

5. a. $7.3 \times 100 = 730$, b. $0.28 \times 100 = 28$, c. $3.9 \times 100 = 390$,
 d. $0.07 \times 100 = 7$ e. $7.36 \times 100 = 736$, f. $2.8 \times 100 = 280$,
 g. $12.63 \times 100 = 1\,263$, h. $0.26 \times 100 = 26$, i. $15.62 \times 100 = 1\,562$,
 j. $0.11 \times 100 = 11$

When a number is multiplied by a hundred, all the digits move two places to the left.

6. When a number is multiplied by a thousand, all the digits move three places to the left.

Page 6

1. Allow a degree or two of inaccuracy in measuring the angles.
 This will mean the totals will not always be exactly 180^0

Triangle	First Angle	Second Angle	Third Angle	Total
A	80^0	60^0	40^0	180^0
B	42^0	85^0	53^0	180^0
C	34^0	90^0	56^0	180^0
D	90^0	43^0	47^0	180^0

The total is 180^0 in each case.

The quadrilateral investigation should reveal that the angles total 360^0

Answers (Contd)

Page 7

1. Here are some examples of what could be said. Accept other sensible answers.

a. rectangle:

Parallelogram with four right angles

Quadrilateral with four right angles

Polygon with four sides and four right angles

b. equilateral triangle:

Triangle with three equal sides

Triangle with three equal angles

Isosceles triangle with the third side equal to the other two

Polygon with three equal sides

Three sided polygon with three equal angles

Triangle with three axes of symmetry

c. kite:

Quadrilateral with pairs of adjacent sides equal

Quadrilateral with an axis of symmetry passing through two opposite angles

d. trapezium:

Quadrilateral with one pair of opposite sides equal

e. parallelogram:

Quadrilateral with two pairs of opposite sides parallel

Quadrilateral with two pairs of opposite sides equal

f. isosceles triangle:

Triangle with two equal sides

Triangle with two equal angles

Triangle with an axis of symmetry

g. regular hexagon:

Hexagon with all sides equal and all angles equal

Polygon with six equal sides and six equal angles

2. $d = 7 \times n$ or $d = 7n$

3. $A = L \times W$ or $A = LW$

4. $P = 2 \times L + 2 \times W$ or $P = 2L + 2W$

5. $2 \times n$ or $2n$

6. $4 \times n$ or $4n$

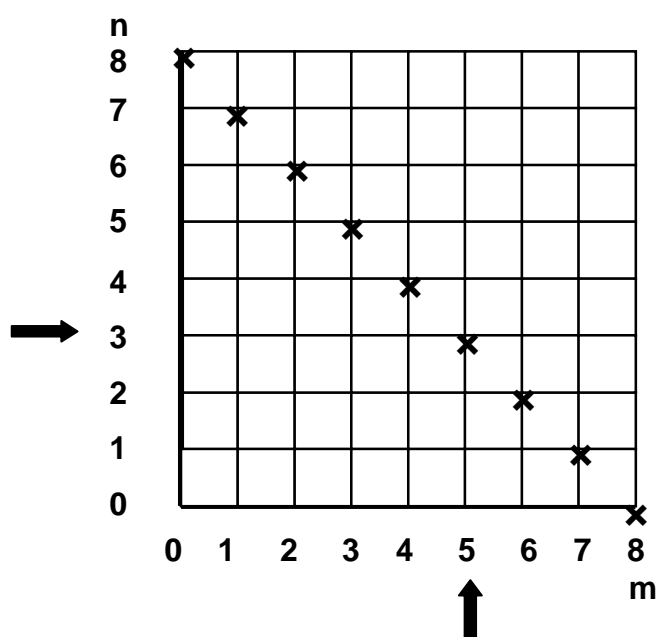
7. $b = 2 \times k \times m$ or $b = 2km$

Answers (Contd)

Page 8

1. $85 \times b$ pence or $85b$ pence
2. $2.4 \times p$ kg or $2.4p$ kg
3. $f + w + (2 \times p) + (2 \times h) + (2 \times t)$ or $f + w + 2p + 2h + 2t$
4. $c + b$ pence; $(3 \times c) + (4 \times b)$ pence or $3c + 4b$ pence

5.



Page 9

1. Name	Edges on base (b)	Faces (f)	Edges (e)	Vertices (v)
Tetrahedron	3	4	6	4
Square based Pyramid	4	5	8	5
Pentagonal based pyramid	5	6	10	6
Hexagonal based pyramid	6	7	12	7
Octagonal based pyramid	8	9	16	9

$$f = b + 1$$

Examples of other possible formulae:

$$v = b + 1, \quad e = 2 \times b, \quad v = f, \quad f = v$$

Some children may discover the following, more difficult, formulae:

$$f + v = e + 2, \quad e = (f - 1) \times 2, \quad e = (v - 1) \times 2$$