CATCHING UP ON NUMERACY: PENCIL AND PAPER METHODS

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THE Chalkface PROJECT

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INTRODUCTION

The pack

Catching up on Numeracy: Pencil and Paper Methods is one of a series of three Chalkface packs that address the concepts, skills and facts within the Number Programme of Study. This pack addresses key skills and concepts in pencil and paper recording. It encourages students to develop a variety of strategies based on known mathematical facts. Subjects covered include squares, factors, multiples, decimals, fractions, and measurement of time, weight and distance.

Student suitability

This pack is for students in Years 7 and 8 working at National Curriculum Levels 3 to 5. It is excellent preparation for the Mental Arithmetic SAT paper which all students must take at the end of Key Stage 3. It is also a useful resource for Year 9 students who need to practise their pencil and paper skills. Because the pack is photocopiable, you can use it flexibly, with individual students, small or large groups, or across a whole year set.

Using the pack

The sheets in the pack may be worked through in any order. The Teachers' Notes opposite each page give its aims, preparation needed, classroom management advice, differentiation possibilities, answers and extension activities. We assume you have access to pen, paper, and chalkboard and are used to teaching in whole class, small group, paired and individual situations. If you have any queries about how best to use the pack, we are happy to help; please write to us at the address below.

Other linked Chalkface packs

Pencil and Paper Methods belongs to the Chalkface series *Catching up on Numeracy*. Other packs in the series are: *Mental Arithmetic* and *Use of Calculators.*

The people involved

David Knock, the consultant and editorial co-ordinator for this pack, is Head of Maths at Elliott Durham School in Mapperley, Nottingham. The pack was written by Tim Bax. The pack was illustrated by Lucy Watson and the cover and series design was by Michael Lopategui. The layout artist and editor was Karen Reed. The Chalkface format was created by Susan Quilliam.

Photocopy laws

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CONTENTS

Lesson-specific Teachers' Notes are to be found on the page facing each worksheet.

— GENERAL GUIDELINES —

The Teachers' Notes opposite each page support the use of each specific page as required. These more general guidelines give advice on using the whole pack. They offer suggestions on preparation, running the lesson and follow-up work, and could form the basis of in-service training prior to using the pack.

Please remember to photocopy both the relevant Teachers' Notes and these General Guidelines if you are copying worksheets for a supply teacher to use.

Preparing for the lesson

- Specific preparation requirements are indicated in the *Preparation* section of the Teachers' Notes. You should always have available copies of the worksheet, pens, pencils and a chalkboard or equivalent.
- Allow approximately an hour's lesson for each page. If there may be too much or too little work for an hour, this is indicated in the *Timing* section of the Teachers' Notes.
- You can link pages to make a double lesson; linkable pages are indicated under the heading *Links*.
- Possible classroom management challenges which may be created by the page and any issues of a sensitive nature are brought to your attention in the Teachers' Notes under the heading *Points To Be Aware Of*. You will probably want to check whether these are relevant to your class.

The lesson

Pages are worded so that you can choose how to manage each in the classroom. However, as a general guideline, we suggest that you move from 'introductory chat' to individual work, through to paired or small group discussion, then to pooling ideas as a class. Where a specific approach is required which differs from this, it is indicated in the Teachers' Notes under the heading *Classroom Management*.

Each sheet contains a number of activities. These fall into several basic formats:

- **Thought starters**
- **Oral work**
- **Research**
- **Written work**
- **Reading**
- **Brainstorming**
- **Working in role or 'imagine' exercises**

Where relevant, you may choose to allow students with poor writing skills to work on the sheet and mark, underline or colour to show understanding.

Where extended writing or copying is required, you could modify the task and set a precise target for students who work very slowly, inaccurately or untidily. You may find it useful to mark sections which you expect students to complete with a fluorescent pen. Where a different approach might be more appropriate for less able (or more able) students, this is highlighted under *Differentiation*.

Following up on the lesson

The Teachers' Notes may include, where relevant, suggestions for *Extension Activities*. These are usually designed to carry the topic into a double lesson, or to provide an opportunity for out-of-classroom work.

– THE NUMERACY FRAMEWORK –

TEACHING PROGRAMME: YEAR 7 (Key objectives are highlighted in bold type.)

NUMBERS AND THE NUMBER SYSTEM

2–7 Place value, ordering and rounding

2–3 • Understand and use decimal notation and place value.

3–4 • Compare and order decimals in different contexts.

4–5 • **Order, add and subtract positive and negative numbers in context.**

6 • Round numbers, including to one and two decimal places.

6–7 • Make and justify estimates and approximations (of numbers and calculations).

8–9 Properties of numbers

8 • Recognise square numbers to at least 12 • 12, the cubes of 1, 2, 3, 4, 5 and 10, and the corresponding roots.

9 • Recognise and use multiples, factors and primes (less than 100); use tests of divisibility.

9 • Write numbers as products of primes, using index notation.

10–17 Fractions, decimals, percentages, ratio and proportion

10–12 • **Use the equivalence of fractions, decimals and percentages in describing proportions** and convert between them (e.g. to order fractions).

13–15 • Find fractions and percentages of quantities.

16–17 • Understand the relationship between ratio and proportion, use ratio and proportion to solve simple problems.

CALCULATIONS

18–19 Number operations and the relationships between them

18–19 • Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction, and of the principles (not the names) of the arithmetic laws.

19 • Know and use the order of operations.

20–25 Mental methods and rapid recall of number facts

20 • Consolidate the rapid recall of number facts, including multiplication facts to 10 • 10, and quickly derive associated division facts.

21–25 • Consolidate and **extend mental methods of calculation to include decimals, fractions and percentages** (accompanied where appropriate by suitable jottings).

26–27 Written methods

26 • Consolidate efficient written methods of addition andsubtraction of whole numbers, and extend to decimals.

26–27 • **Refine written methods of multiplication and division of whole numbers to ensure efficiency, and extend to decimals with two places.**

28 Calculator methods

28 • Plan and carry out calculations using the facilities on a calculator, including the square root and percentage keys, the memory and brackets.

28 • Interpret the display on a calculator in different contexts (fractions, decimals, money, metric measures, time).

29 Checking results

29 • Judge whether an answer is reasonable and check results, including using:
– knowledge of the number system;
– rounding to approximate;
– inverse operations.

SOLVING PROBLEMS

30–38 Solving problems

30–35 • Solve problems and puzzles in a variety of contexts (number, algebra, shape, space and measures).

36 • **Choose and justify the use of an appropriate and efficient method for solving a problem.**

37 • Explain methods and reasoning, orally and in writing.

38 • Predict, generalise and suggest extensions by asking 'What if …?'

ALGEBRA

39–43 Equations and formulae

39 • **Use letters or symbols to represent unknown numbers or variables.**

40 • **Know that algebraic operations follow the same conventions and order as arithmetic operations.**

40 • Simplify linear algebraic expressions by collecting like terms; begin to multiply a single term over a bracket.

41–42 • Use formulae from mathematics and other subjects, substitute numbers in simple formulae and, in simple cases, derive a formula

42–43 • Construct and solve simple linear equations, selecting an appropriate method.

44–49 Sequences and functions

44 • Generate and describe in words common integer sequences, and sequences from practical contexts.

45–46 • **Generate terms of a sequence, given a rule (e.g. finding one term from the previous term, finding a term given its position in the sequence).**

47 • Describe the general term of a simple sequence in words, then using symbols.

48–49 • Express simple functions in words, then using symbols.

50–52 Graphs

50 • **Find co-ordinate pairs that satisfy a rule and plot these on a co-ordinate grid.**

50–51 • Recognise that a function such as $y = 3x + 7$ corresponds to a straight-line graph.

51–52 • Begin to plot the graphs of linear functions arising from real-life problems; discuss and interpret a range of graphs arising from real situations.

SHAPE, SPACE AND MEASURES

53-55 Lines and angles

53 • Use accurately the vocabulary and notation associated with lines and angles.

53-55 • **Recognise and use parallel lines and the sum of angles at a point, on a straight line and in triangles.**

56-59 Properties of shapes

56 • Visualise, describe and sketch 2-D shapes in different orientations.

57-58 • Use the geometric properties of triangles and quadrilaterals.

58-59 • Visualise and describe 3-D shapes from 2-D representations.

60-64 Transformations

60 • Understand and use the language and notation associated with reflections, translations and rotations.

60-61 • Reflect 2-D shapes in given mirror lines, and recognise line symmetry.

62 • Translate 2-D shapes.

63-64 • Rotate 2-D shapes about a given point, and recognise rotational symmetry.

65 Co-ordinates

65 • Consolidate use of the conventions and notation for 2-D co-ordinates in all four quadrants.

65 • Find co-ordinates of points determined by geometric information.

66 Construction

66 • Consolidate measuring and drawing:
– lines to the nearest millimetre;
– angles to the nearest degree, and extend to reflex angles.

66 • Construct triangles and other 2-D shapes, using a ruler and protractor.

67-71 Measures

67 • Use names and abbreviations of metric and imperial units for estimation, measurement, calculation and problem solving in contexts involving length, area, mass, capacity and time.

68 • **Convert from one metric unit to another** (e.g. grams to kilograms).

68 • Know rough metric equivalents of imperial measures in common use (feet, miles, pounds, ounces, pints, gallons).

68 • **Read and interpret scales on a range of measuring instruments.**

69 • Calculate the perimeter and area of compound shapes made up of rectangles.

70-71 • Calculate the surface area of cuboids and compound shapes made from cuboids.

HANDLING DATA

72-73 Specifying a problem, planning and collecting data

72 • Respond to a given problem, and predict and hypothesise about possible answers.

72 • Identify which data need to be collected and how.

73 • Collect data from surveys, experiments and secondary sources, and record in a frequency table, grouped where appropriate in equal class intervals.

74-75 Processing data

74-75 • Calculate statistics from data, using ICT as appropriate:
– find the mode of a small data set;
– calculate the mean of a set of discrete data, using a calculator for a large number of items;
– find the median of a small number of items;
– find and use the range of a set of values.

76-79 Representing data, and interpreting and discussing results

76 • Construct graphs and diagrams to represent data (e.g. bar-line graph, frequency diagram for a discrete variable).

77 • Use ICT to produce graphs and charts, and identify which are most useful in the context of the problem.

78 • Interpret diagrams and graphs (including pie charts), and draw inferences based on the shape of graphs and simple statistics for a single distribution.

79 • **Compare two simple distributions using the range and one of the measures of average.**

80-84 Probability

80 • Use vocabulary and ideas of probability, drawing on experience

81 • Recognise that probability is a way of measuring chance or likelihood; **know that probabilities lie between 0 and 1, and calculate probabilities based on equally likely outcomes in simple contexts.**

82 • Identify all possible outcomes of an experiment.

83 • Collect experimental data and record in a frequency table, and estimate probabilities based on the data.

84 • Compare experimental and theoretical probabilities in simple contexts.

Teachers' Notes
DIFFERENT WAYS

Aims Working through plenty of practical examples, students see various equally effective ways to multiply numbers.

Preparation You should photocopy the bingo sheet (see APPENDIX, page 62) for distribution for students in Activity 3.

Activity 1 **Grouping:** Groups of five or six. There should be an even number of groups so that they can compare results – this could lead into a brief classroom discussion of all methods of multiplication.

Answers:

28 x 2 = 56	32 x 2 = 64	28 x 9 = 252	43 x 5 = 215
34 x 5 = 170	21 x 5 = 105	53 x 2 = 106	47 x 8 = 376

Activity 3 **Grouping:** The bingo game should be undertaken on an individual basis. While students are completing the exercise, you should write the following method up on the board:

29 x 5 = 20 x 5 + 9 x 5
= 100 + 45
= 145

Activities 4 and 5 **Grouping:** Students should work on these sums individually, coming together in pairs at the end of the lesson to check each other's answers.

Answers (Activity 4):

33 x 5 = 165	32 x 7 = 224	56 x 2 = 112

Answers (Activity 5):

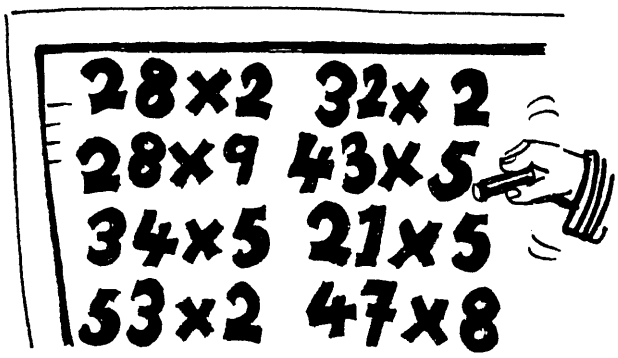
a) 37 x 3 = 111	b) 12 x 32 = 384	c) 41 x 32 = 1312
d) 35 x 12 = 420	e) 23 x 1 = 23	f) 23 x 10 = 230

NC Level Number and Algebra – 3

DIFFERENT WAYS

1 In your group, work out the answers to the eight sums on the right.

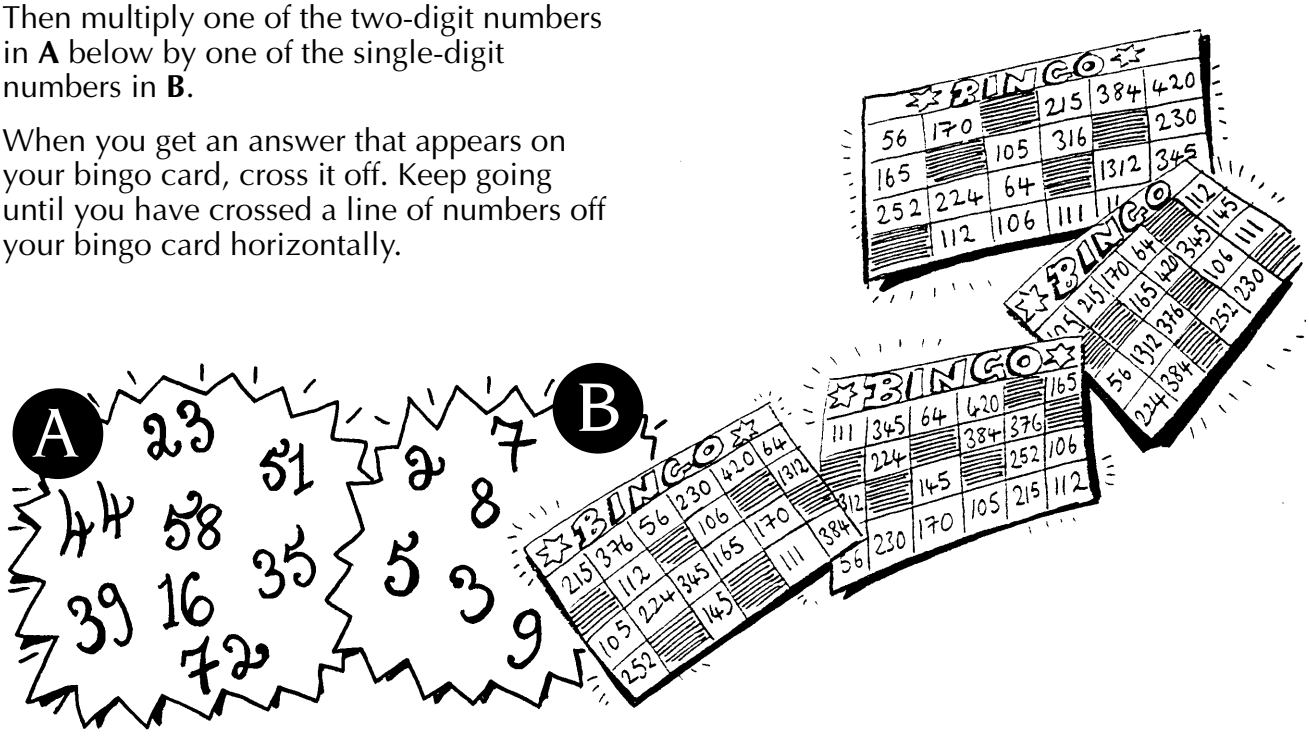
2 When you have all the answers, compare them with another group's. Did you get the same answers? Did you use the same methods? Which method do you prefer?



3 Now you are going to play multiplication bingo. Choose one of the bingo cards from the sheet your teacher will give you.

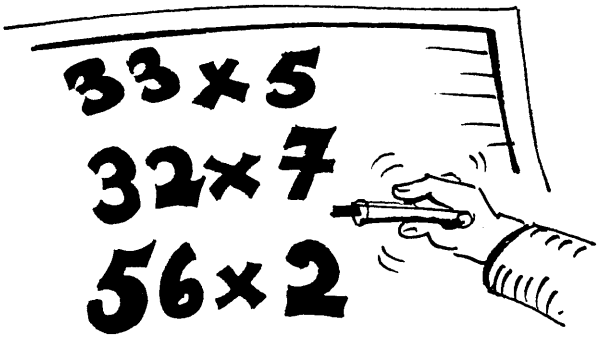
Then multiply one of the two-digit numbers in **A** below by one of the single-digit numbers in **B**.

When you get an answer that appears on your bingo card, cross it off. Keep going until you have crossed a line of numbers off your bingo card horizontally.



4 Look at the method your teacher has put on the board. Use it to answer the three sums on the right. Check your answers using another method.

5 Now answer the questions below using your favourite method.



EXAM PAPER

1a) 37×3 b) 12×32 c) 41×32

d) 35×12 e) 23×1 f) 23×10

Teachers' Notes
FIVES AND TENS

Aims This sheet asks students to investigate the properties and uses of 'multiplication chains'.

Classroom Management This lesson should take place in pairs, so that students can compare results. You should patrol the classroom, giving advice and encouragement where required.

Activity 1 **Answers:**
All the multiples produced should end in 0.

Activity 3 **Answers:**

48 x 5 = 240
32 x 5 = 160
44 x 5 = 220
37 x 5 = 185

242 x 5 = 1210
222 x 5 = 1110
27 x 5 = 135

The answers are all half of the original number, times 10.
A number multiplied by 5 is half of the same number multiplied by 10.

Activity 4 **Answers:**

284 x 5 = 1 420
151 x 5 = 755
1234 x 5 = 6 170

326 x 5 = 1 630
27 x 5 = 135

284 x 50 = 14 200
151 x 50 = 7 550
1234 x 50 = 61 700

326 x 50 = 16 300
27 x 50 = 1 350

284 x 500 = 142 000
151 x 500 = 75 500
1234 x 500 = 617 000

326 x 500 = 163 000
27 x 500 = 13 500

NC Level Number and Algebra – 3