CATCHING UP ON NUMERACY: MENTAL ARITHMETIC

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THE Chalkface PROJECT

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INTRODUCTION

The pack

Catching up on Numeracy: Mental Arithmetic is one of a series of three packs which identify the concepts, skills and facts required within the Numeracy strand Ma2. This pack addresses key skills and concepts in simple stages, and encourages students to develop a variety of strategies based on 'known' mathematical facts.

Student suitability

The pack is written for use with students in years 7 and 8, working at National Curriculum Levels 3 to 5. It prepares them for the Mental Arithmetic SAT paper, that became compulsory in 1998 at the end of Key Stage 3, and is therefore also suitable for Year 9 students needing to boost their skills.

Using the pack

Each page provides a stand-alone lesson, with one or more core activities and a definite and identifiable conclusion for students to reach. Regular practice of mental arithmetic skills is the most effective way to consolidate them, and the pack provides opportunities for students to work with a wide range of required mathematical skills. It is recommended that a page a week from this pack is included in students' mathematical work. Activities on all the pages are designed to be attempted mentally. Answers can be recorded individually, with pencil and paper; given orally, or 'collected' on the chalkboard. Working in groups may be a useful approach for less able students attempting some of the more difficult activities. Full answers are given on every page of Teachers' Notes opposite each worksheet, and offer the option of students marking their own work. The Teachers' Notes also give the page's aims, preparation needed, classroom management advice, differentiation possibilities, and extension activities. We presume that you have access to pen, paper and chalkboard, and that students are used to working both individually and in small and large discussion groups. If you have a query about how best to use the pack, we are happy to help; please write to us at the address below.

Added value

The mental arithmetic activities on every page become progressively longer and more challenging. As students work through the page, their skills are tested, enhanced and stretched.

Other linked Chalkface packs

This pack is one of a series of three Catching up on Numeracy packs. The others are: Use of Calculators,and Pencil and Paper Methods.

The people involved

David Saunders, the consultant author for this pack, is Head of Mathematics at Cullompton Community College. Shaun McCarthy is a freelance writer. The pack was illustrated by Steve Tatler and the cover and series design was by Michael Lopategui. Patricia Maxwell was the editorial co-ordinator, Katherine Benzinski was the editor, and Karen Reed was the layout artist. The pack was printed by Prontaprint Barking and The Modern Printers. The Chalkface format was created by Susan Quilliam.

Photocopy laws

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CONTENTS

CONTENTS

GENERAL GUIDELINES

The Teachers' Notes opposite each page support the use of each specific page as required. These more general guidelines give advice on using the whole pack. They offer suggestions on preparation, running the lesson and follow-up work, and could form the basis of in-service training prior to using the pack.

Please remember to photocopy both the relevant Teachers' Notes and these General Guidelines if you are copying worksheets for a supply teacher to use.

Preparing for the lesson

- Specific preparation requirements are indicated in the *Preparation* section of the Teachers' Notes. You should always have available copies of the worksheet, pens, pencils and a chalkboard or equivalent.
- Allow approximately an hour's lesson for each page. If there may be too much or too little work for an hour, this is indicated in the *Timing* section of the Teachers' Notes.
- You can link pages to make a double lesson; linkable pages are indicated under the heading *Links*.
- Possible classroom management challenges which may be created by the page are flagged in the Teachers' Notes under the heading *Points To Be Aware Of*, and any issues of a sensitive nature are brought to your attention under *Sensitivity*. You will probably want to check whether these are relevant to your class.

The lesson

Pages are worded so that you can choose how to manage each in the classroom. However, as a general guideline, we suggest that you move from 'introductory chat' to individual work, through to paired or small group discussion, then to pooling ideas as a class. Where a specific approach is required which differs from this, it is indicated in the Teachers' Notes under the heading *Approach*.

Each sheet contains a number of activities. These fall into several basic formats:

- **Thought starters**
- **Oral work**
- **Research**
- **Written work**
- **Reading**
- **Brainstorming**
- **Working in role or 'imagine' exercises**

Where relevant, you may choose to allow students with poor writing skills to work on the sheet and mark, underline or colour to show understanding.

Where extended writing or copying is required, you could modify the task and set a precise target for students who work very slowly, inaccurately or untidily. You may find it useful to mark sections which you expect students to complete with a fluorescent pen. Where a different approach might be more appropriate for less able (or more able) students, this is highlighted under *Differentiation*.

Following up on the lesson

The Teachers' Notes may include, where relevant, suggestions for *Extension Activities*. These are usually designed to carry the topic into a double lesson, or to provide an opportunity for out-of-classroom work.

– THE NUMERACY FRAMEWORK –

TEACHING PROGRAMME: YEAR 7 (Key objectives are highlighted in bold type.)

NUMBERS AND THE NUMBER SYSTEM

2–7 Place value, ordering and rounding

2–3 • Understand and use decimal notation and place value.

3–4 • Compare and order decimals in different contexts.

4–5 • **Order, add and subtract positive and negative numbers in context.**

6 • Round numbers, including to one and two decimal places.

6–7 • Make and justify estimates and approximations (of numbers and calculations).

8–9 Properties of numbers

8 • Recognise square numbers to at least 12 • 12, the cubes of 1, 2, 3, 4, 5 and 10, and the corresponding roots.

9 • Recognise and use multiples, factors and primes (less than 100); use tests of divisibility.

9 • Write numbers as products of primes, using index notation.

10–17 Fractions, decimals, percentages, ratio and proportion

10–12 • **Use the equivalence of fractions, decimals and percentages in describing proportions** and convert between them (e.g. to order fractions).

13–15 • Find fractions and percentages of quantities.

16–17 • Understand the relationship between ratio and proportion, use ratio and proportion to solve simple problems.

CALCULATIONS

18–19 Number operations and the relationships between them

18–19 • Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction, and of the principles (not the names) of the arithmetic laws.

19 • Know and use the order of operations.

20–25 Mental methods and rapid recall of number facts

20 • Consolidate the rapid recall of number facts, including multiplication facts to 10 • 10, and quickly derive associated division facts.

21–25 • Consolidate and **extend mental methods of calculation to include decimals, fractions and percentages** (accompanied where appropriate by suitable jottings).

26–27 Written methods

26 • Consolidate efficient written methods of addition andsubtraction of whole numbers, and extend to decimals.

26–27 • **Refine written methods of multiplication and division of whole numbers to ensure efficiency, and extend to decimals with two places.**

28 Calculator methods

28 • Plan and carry out calculations using the facilities on a calculator, including the square root and percentage keys, the memory and brackets.

28 • Interpret the display on a calculator in different contexts (fractions, decimals, money, metric measures, time).

29 Checking results

29 • Judge whether an answer is reasonable and check results, including using:
– knowledge of the number system;
– rounding to approximate;
– inverse operations.

SOLVING PROBLEMS

30–38 Solving problems

30–35 • Solve problems and puzzles in a variety of contexts (number, algebra, shape, space and measures).

36 • **Choose and justify the use of an appropriate and efficient method for solving a problem.**

37 • Explain methods and reasoning, orally and in writing.

38 • Predict, generalise and suggest extensions by asking 'What if …?'

ALGEBRA

39–43 Equations and formulae

39 • **Use letters or symbols to represent unknown numbers or variables.**

40 • **Know that algebraic operations follow the same conventions and order as arithmetic operations.**

40 • Simplify linear algebraic expressions by collecting like terms; begin to multiply a single term over a bracket.

41–42 • Use formulae from mathematics and other subjects, substitute numbers in simple formulae and, in simple cases, derive a formula

42–43 • Construct and solve simple linear equations, selecting an appropriate method.

44–49 Sequences and functions

44 • Generate and describe in words common integer sequences, and sequences from practical contexts.

45–46 • **Generate terms of a sequence, given a rule (e.g. finding one term from the previous term, finding a term given its position in the sequence).**

47 • Describe the general term of a simple sequence in words, then using symbols.

48–49 • Express simple functions in words, then using symbols.

50–52 Graphs

50 • **Find co-ordinate pairs that satisfy a rule and plot these on a co-ordinate grid.**

50–51 • Recognise that a function such as $y = 3x + 7$ corresponds to a straight-line graph.

51–52 • Begin to plot the graphs of linear functions arising from real-life problems; discuss and interpret a range of graphs arising from real situations.

SHAPE, SPACE AND MEASURES

53-55 Lines and angles

53 • Use accurately the vocabulary and notation associated with lines and angles.

53-55 • **Recognise and use parallel lines and the sum of angles at a point, on a straight line and in triangles.**

56-59 Properties of shapes

56 • Visualise, describe and sketch 2-D shapes in different orientations.

57-58 • Use the geometric properties of triangles and quadrilaterals.

58-59 • Visualise and describe 3-D shapes from 2-D representations.

60-64 Transformations

60 • Understand and use the language and notation associated with reflections, translations and rotations.

60-61 • Reflect 2-D shapes in given mirror lines, and recognise line symmetry.

62 • Translate 2-D shapes.

63-64 • Rotate 2-D shapes about a given point, and recognise rotational symmetry.

65 Co-ordinates

65 • Consolidate use of the conventions and notation for 2-D co-ordinates in all four quadrants.

65 • Find co-ordinates of points determined by geometric information.

66 Construction

66 • Consolidate measuring and drawing:
– lines to the nearest millimetre;
– angles to the nearest degree, and extend to reflex angles.

66 • Construct triangles and other 2-D shapes, using a ruler and protractor.

67-71 Measures

67 • Use names and abbreviations of metric and imperial units for estimation, measurement, calculation and problem solving in contexts involving length, area, mass, capacity and time.

68 • **Convert from one metric unit to another** (e.g. grams to kilograms).

68 • Know rough metric equivalents of imperial measures in common use (feet, miles, pounds, ounces, pints, gallons).

68 • **Read and interpret scales on a range of measuring instruments.**

69 • Calculate the perimeter and area of compound shapes made up of rectangles.

70-71 • Calculate the surface area of cuboids and compound shapes made from cuboids.

HANDLING DATA

72-73 Specifying a problem, planning and collecting data

72 • Respond to a given problem, and predict and hypothesise about possible answers.

72 • Identify which data need to be collected and how.

73 • Collect data from surveys, experiments and secondary sources, and record in a frequency table, grouped where appropriate in equal class intervals.

74-75 Processing data

74-75 • Calculate statistics from data, using ICT as appropriate:
– find the mode of a small data set;
– calculate the mean of a set of discrete data, using a calculator for a large number of items;
– find the median of a small number of items;
– find and use the range of a set of values.

76-79 Representing data, and interpreting and discussing results

76 • Construct graphs and diagrams to represent data (e.g. bar-line graph, frequency diagram for a discrete variable).

77 • Use ICT to produce graphs and charts, and identify which are most useful in the context of the problem.

78 • Interpret diagrams and graphs (including pie charts), and draw inferences based on the shape of graphs and simple statistics for a single distribution.

79 • **Compare two simple distributions using the range and one of the measures of average.**

80-84 Probability

80 • Use vocabulary and ideas of probability, drawing on experience

81 • Recognise that probability is a way of measuring chance or likelihood; **know that probabilities lie between 0 and 1, and calculate probabilities based on equally likely outcomes in simple contexts.**

82 • Identify all possible outcomes of an experiment.

83 • Collect experimental data and record in a frequency table, and estimate probabilities based on the data.

84 • Compare experimental and theoretical probabilities in simple contexts.

Teachers' Notes
COUNT OUT LOUD

Aims This page helps students to develop their mental agility through practice, and suggests imagining movement along a number line in regular steps as an aid to developing mental arithmetic skills.

Preparation A number line is a very useful resource for many students and can be used, in conjunction with counting, to encourage visualisation techniques.

Answers

3)

15	+ 10	= 25 and then + 5	= **30**
and then	+ 10	= 40 and then + 5	= **45**
and then	+ 10	= 55 and then + 5	= **60**
and then	+ 10	= 70 and then + 5	= **75**
and then	+ 10	= 85 and then + 5	= **90**

4) 15 x 8 = 120

5) Four 15s in one hour

6) a) a one and a half hour football match = 6 15s (90 min)
b) a two hour train ride = 8 15s (120 min)
c) a three and a half hour plane journey = 14 15s (210 min)
d) a four and a quarter hour delay at an airport = 17 15s (255 min)
e) a six and a quarter hour game of Monopoly = 25 15s (375 min)

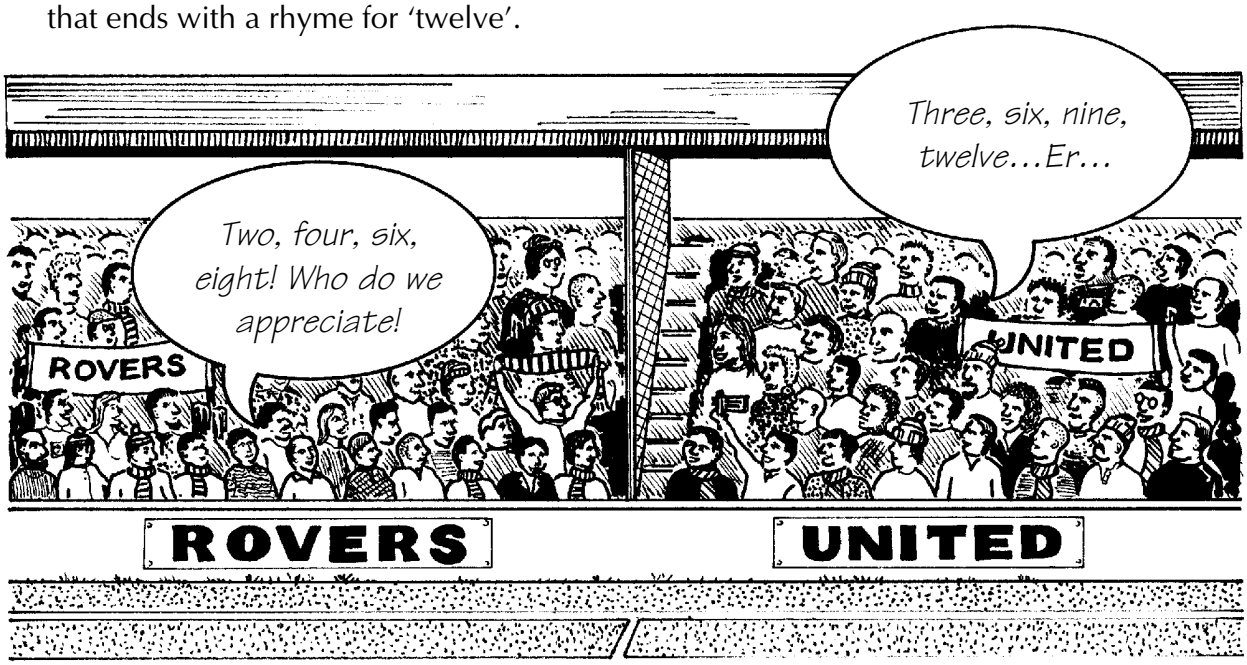
All the counting and sequencing activities involving 15s can be repeated with 12s or 'dozens'. Examples from everyday life (instead of the rugby tour question) can be used: eggs are an obvious example; 'If a crate contains 144 eggs, how many dozens is that?'

Extension Activities Students can practise counting smoothly in 12s up to 144, then continue up to 288.

COUNT OUT LOUD

1) Rovers fans are counting in twos in their chant. United are counting in threes. Unfortunately, they haven't thought up a line to follow their numbers. Invent one that ends with a rhyme for 'twelve'.

2) Practise counting in twos and then threes – up to 30 – until you can do this quickly and with no mistakes.



3) Counting in 15s is a bit more tricky. Think of 15 as a ten and a five. Look at these figures. Copy them out and extend the sequence up to 90.

15 + 10 = 25 and then + 5 = 30…

4) Look at the picture, and answer the rugby tour organiser's question.

5) How many 15s are there in one hour? (Remember that 15 minutes equals a quarter of an hour.)

6) Counting in 15s, work out how many minutes there are in:

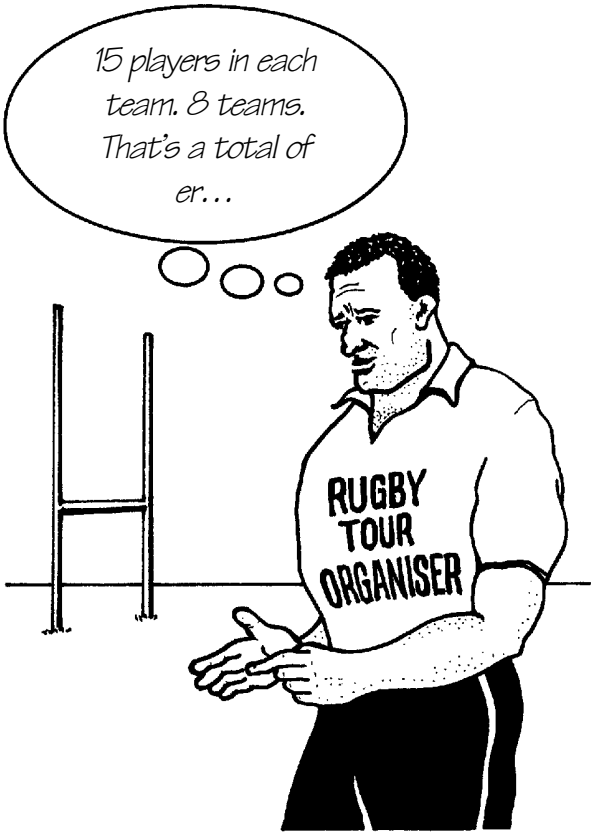
a) a one and a half hour football match

b) a two hour train ride

c) a three and a half hour plane journey

d) a four and a quarter hour delay at an airport

e) a six and a quarter hour game of Monopoly.



Teachers' Notes
BONDS

Aims By practising number bonds up to ten, and then extending this to those for 100 and 1000, this page aims to encourage students to recognise pairs that bond. This will then aid them in the mental addition of large numbers.

Classroom Management For the activity where students work out number bonds that add up to 100, it may be advisable to set a time limit, and see how many they can come up with (perhaps 30 in 10 minutes).

Answers

1) Examples would be: 2 + 8; 6 + 4; 9 + 1

2) Examples would be: 1 + 2 + 7; 2 + 3 + 5

3) 'Agent Bond' would see: 400 + 600 = 1000 and 500 + 500 = 1000

4)
$$\begin{aligned} 30 + 60 &= 90 \\ 1 + 9 &= 10 \\ &= 100 \end{aligned}$$

Extension Activities Students could work in pairs to complete further number bonds. Each student could work out ten number bonds for numbers that add up to 100 or 1000. They should write these out, but omit the second number in the bond, swapping with other students, for them to find the missing number. The calculations should be done mentally.